



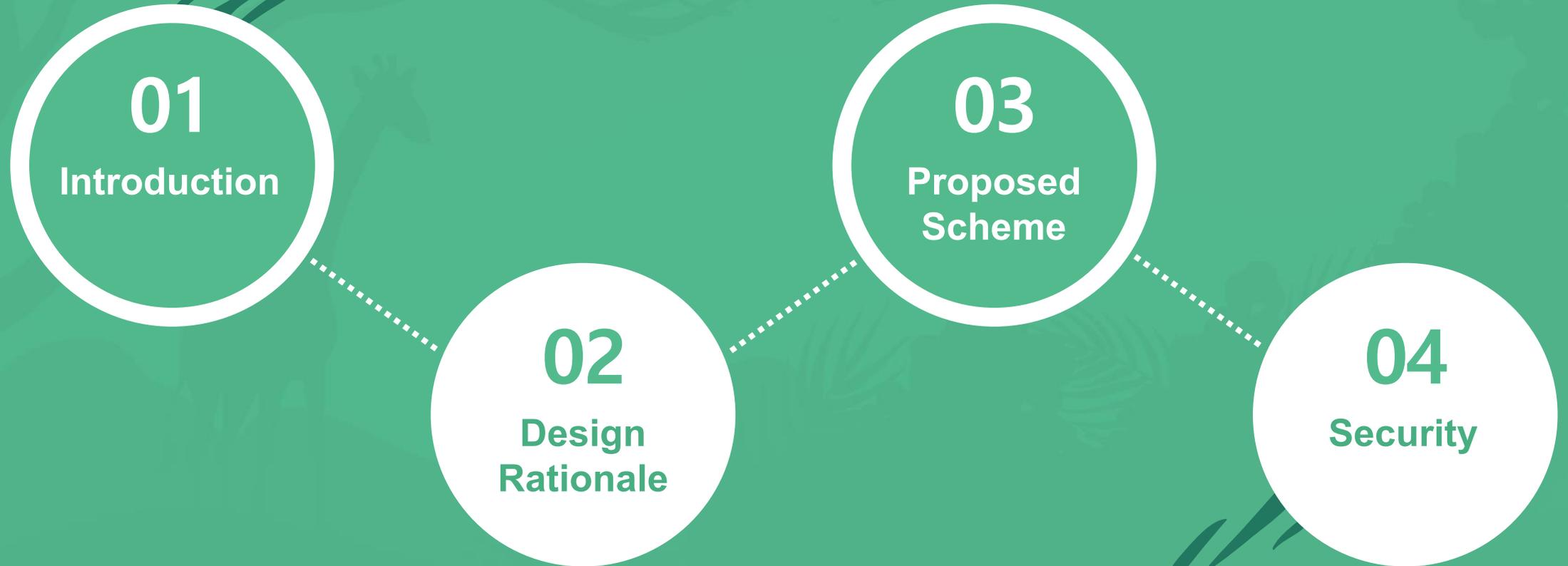
TIGER

Tiny bandwidth KEM
for easy mi**G**ration
based on RLWE**(R)**

Seunghwan Park, Chi-Gon Jung, Aesun Park,
Joongeun Choi, and Honggoo Kang



Contents



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Introduction



Post-Quantum Cryptography

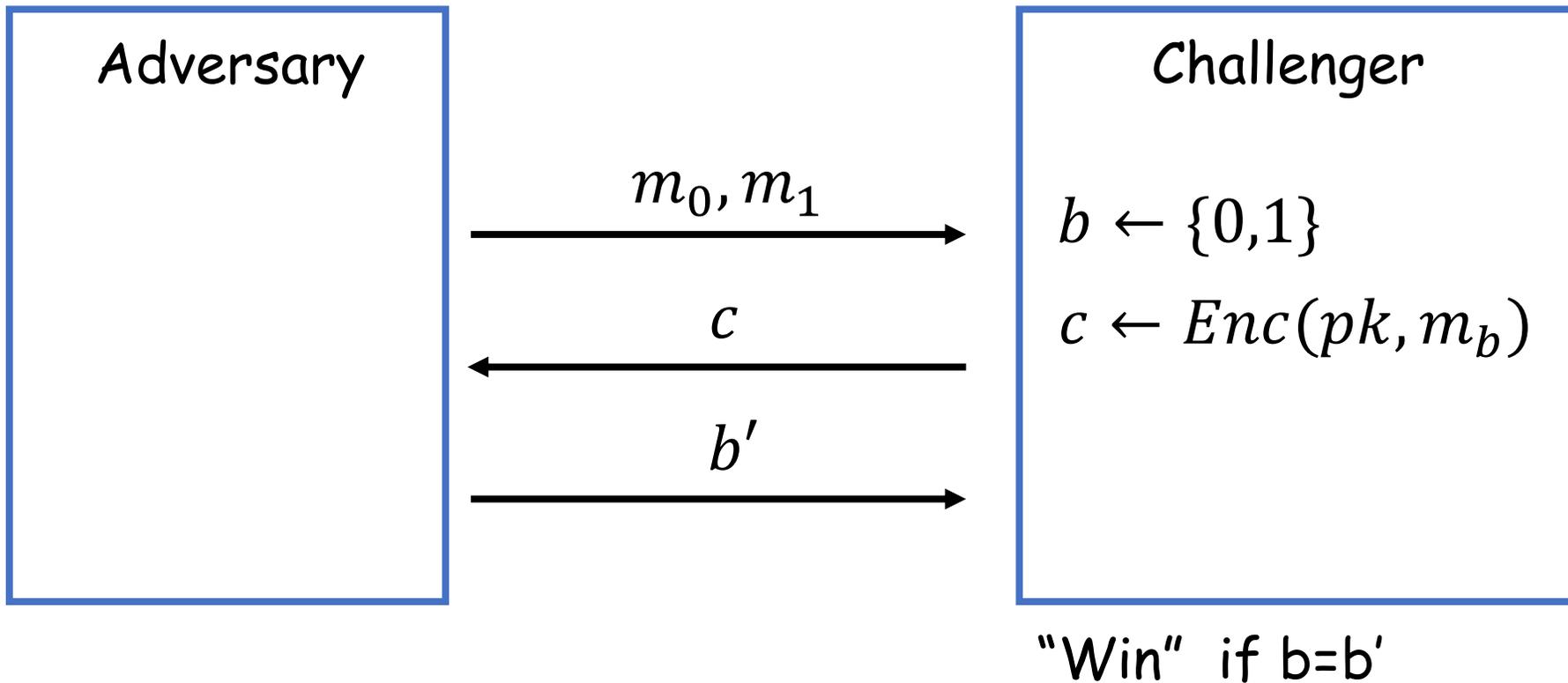
Post-Quantum Cryptography

- Lattice-based
- Code-based
- Multi-variate
- Hash-based
- Isogeny-based

Background

□ Provable Security

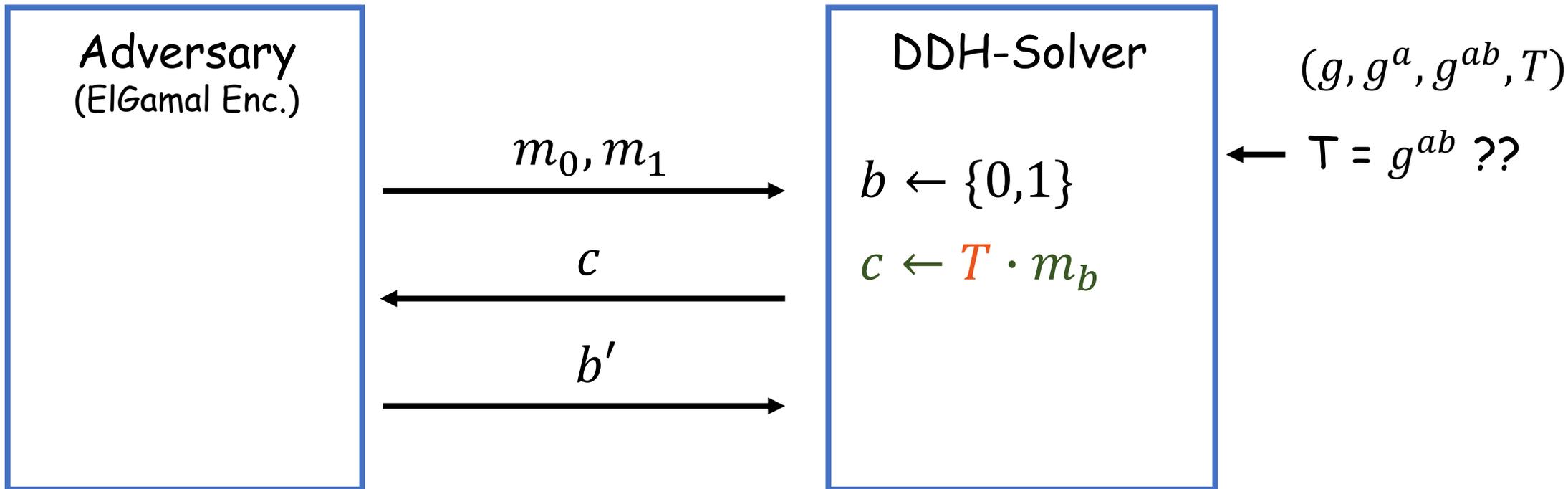
- Hard Problem & Public Key Encryption



Background

□ Provable Security

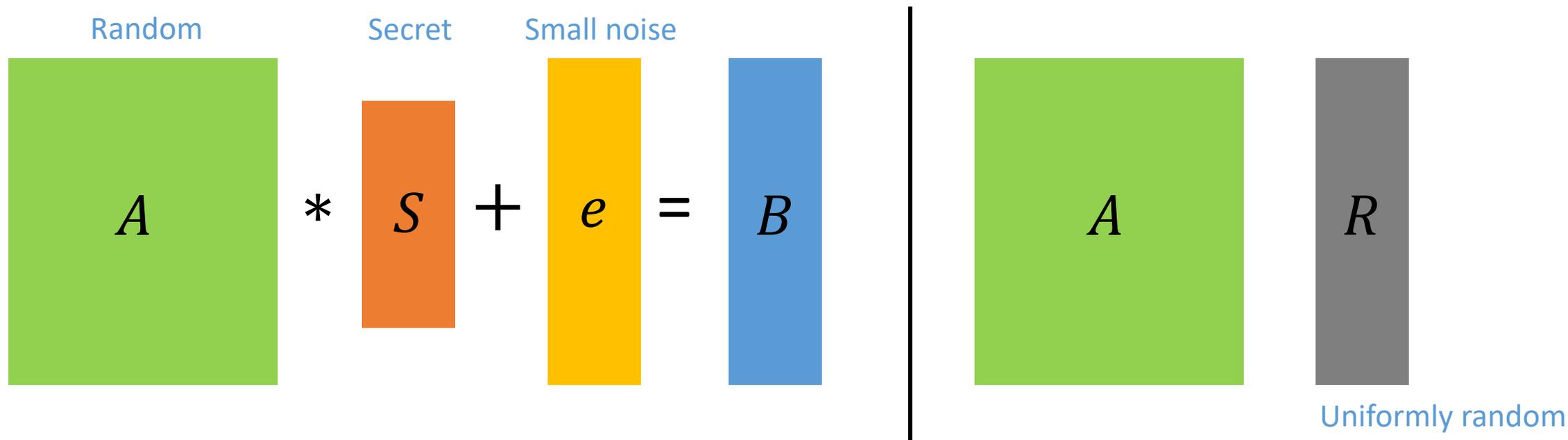
- Hard Problem & Public Key Encryption



Background

Learning-With-Errors(LWE)

- decisional LWE

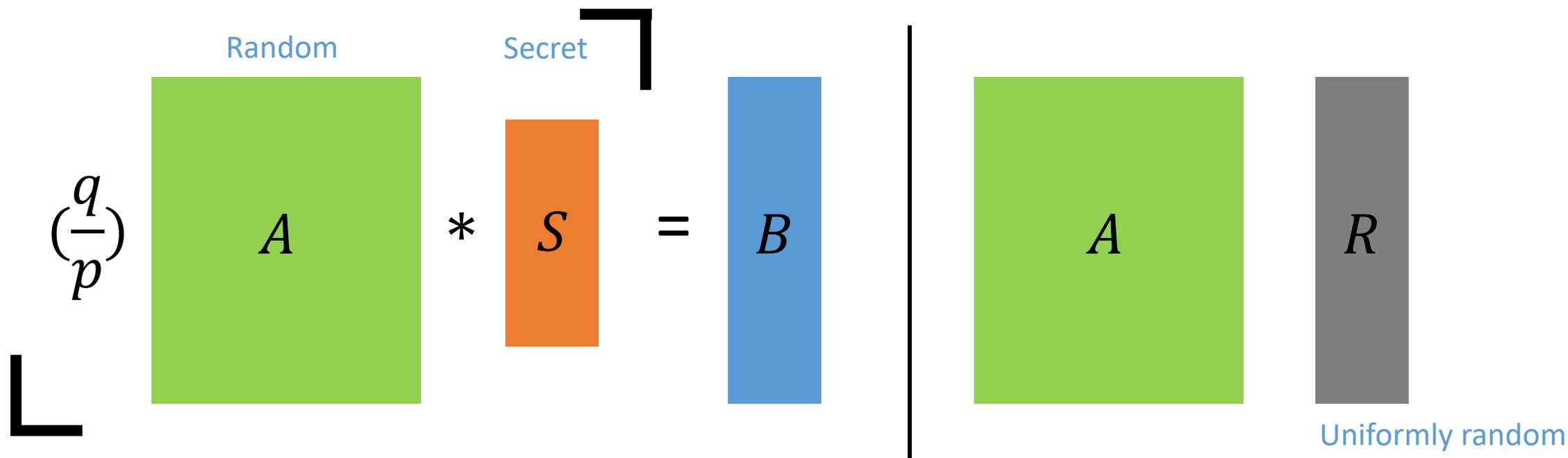


Distinguish (A, B) from (A, R)

Background

□ Learning-With-Rounding(LWR)

- decisional LWR

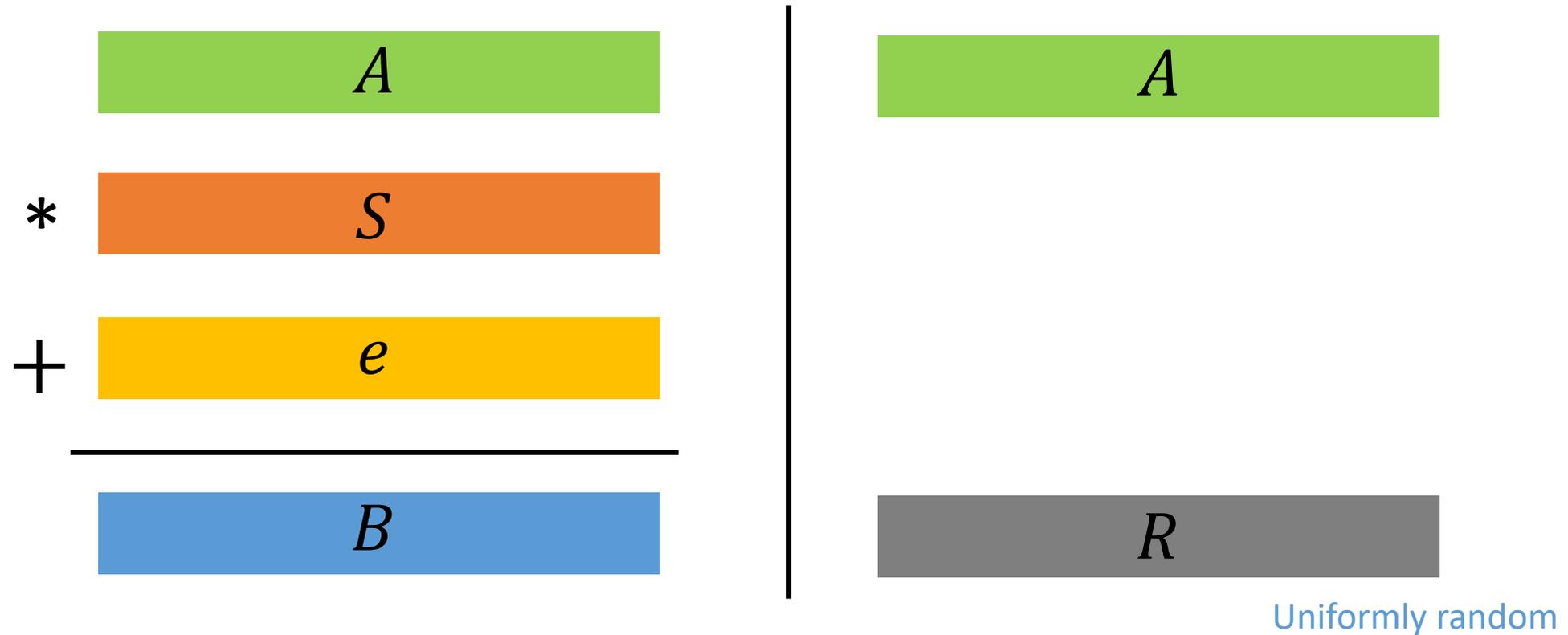


Distinguish (A, B) from (A, R)

Background

□ Learning-With-Errors(LWE)

- decisional Ring-LWE(RLWE)



Distinguish (A, B) from (A, R)

Background

□ Learning-With-Errors(LWE)

- decisional Ring-LWE(RLWE)

$$q \in \mathbb{Z}, R_q := R/qR = \mathbb{Z}_q[X]/\langle X^4 + 1 \rangle$$

$$10x^3 + 11x^2 + 11x + 4$$

$$* \quad 11x^3 + 11x^2 + 9x + 6$$

$$+ \quad 1x^3 + 1x^2 - 1x + 0$$

$$7x^3 + 10x^2 + 5x + 10$$

$$10x^3 + 11x^2 + 11x + 4$$

$$r_3x^3 + r_2x^2 + r_1x + r_0$$

Uniformly random

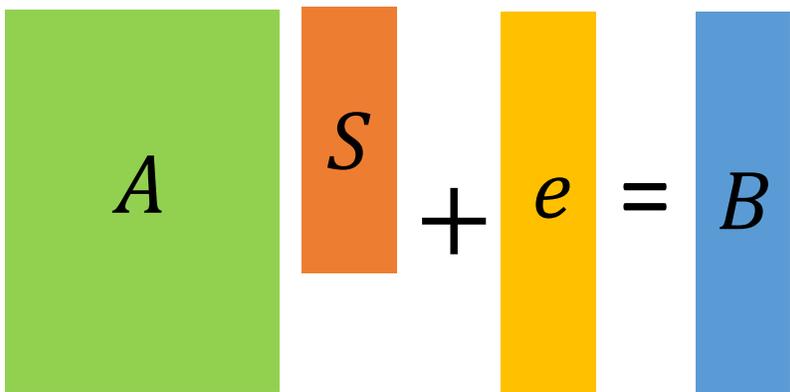
Distinguish (A, B) from (A, R)

Background

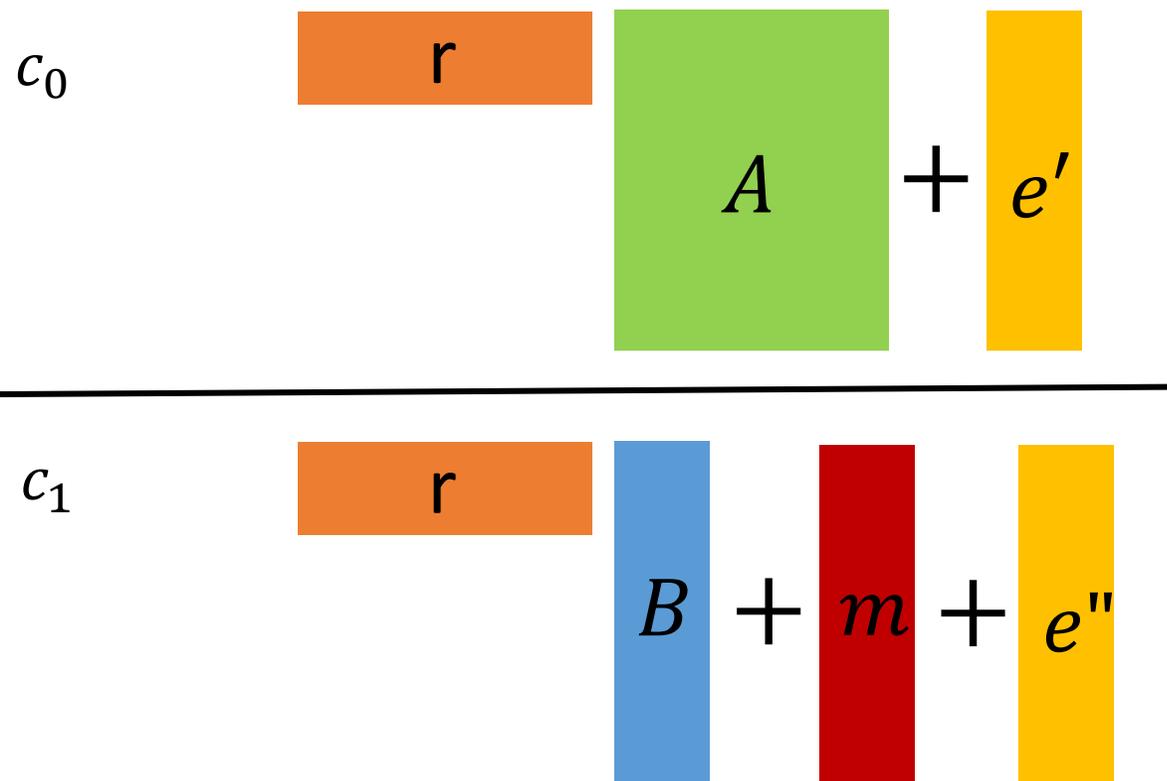
□ CPA-secure Public-Key Encryption (PKE)

- [LP11]

$KeyGen(1^\lambda) \rightarrow pk = \langle A, B \rangle$
 $sk = \langle S \rangle$



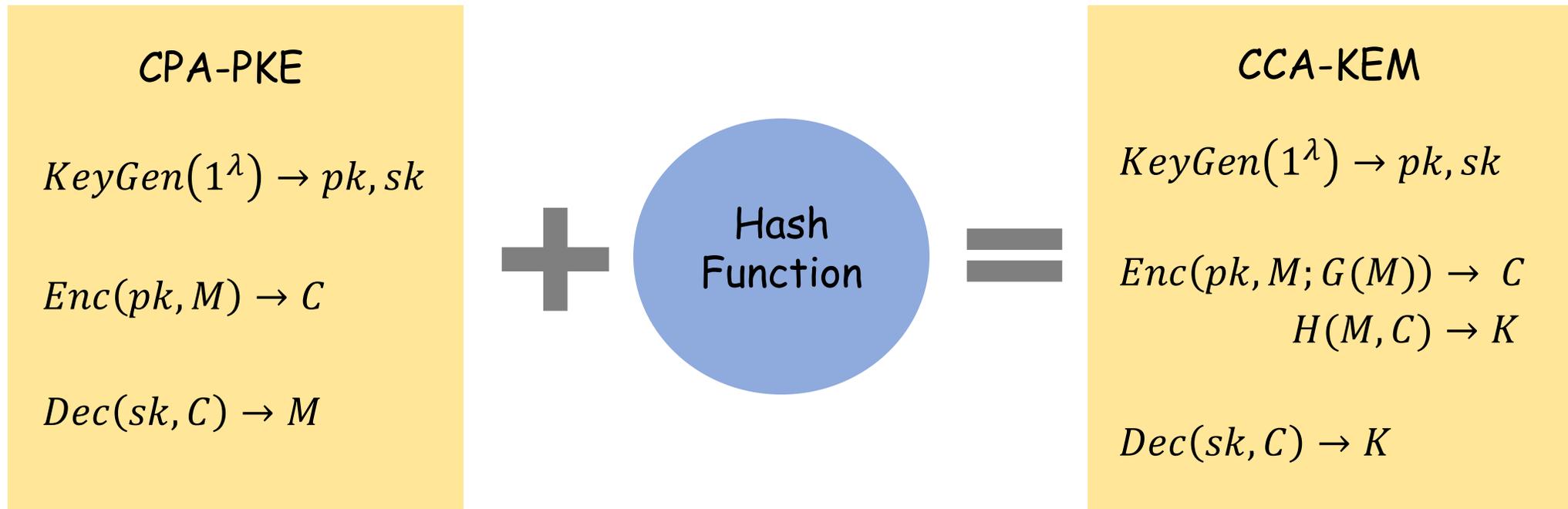
$Encryption(pk, m) \rightarrow c = \langle c_0, c_1 \rangle$



Background

□ CCA-secure Key encapsulation mechanism (KEM)

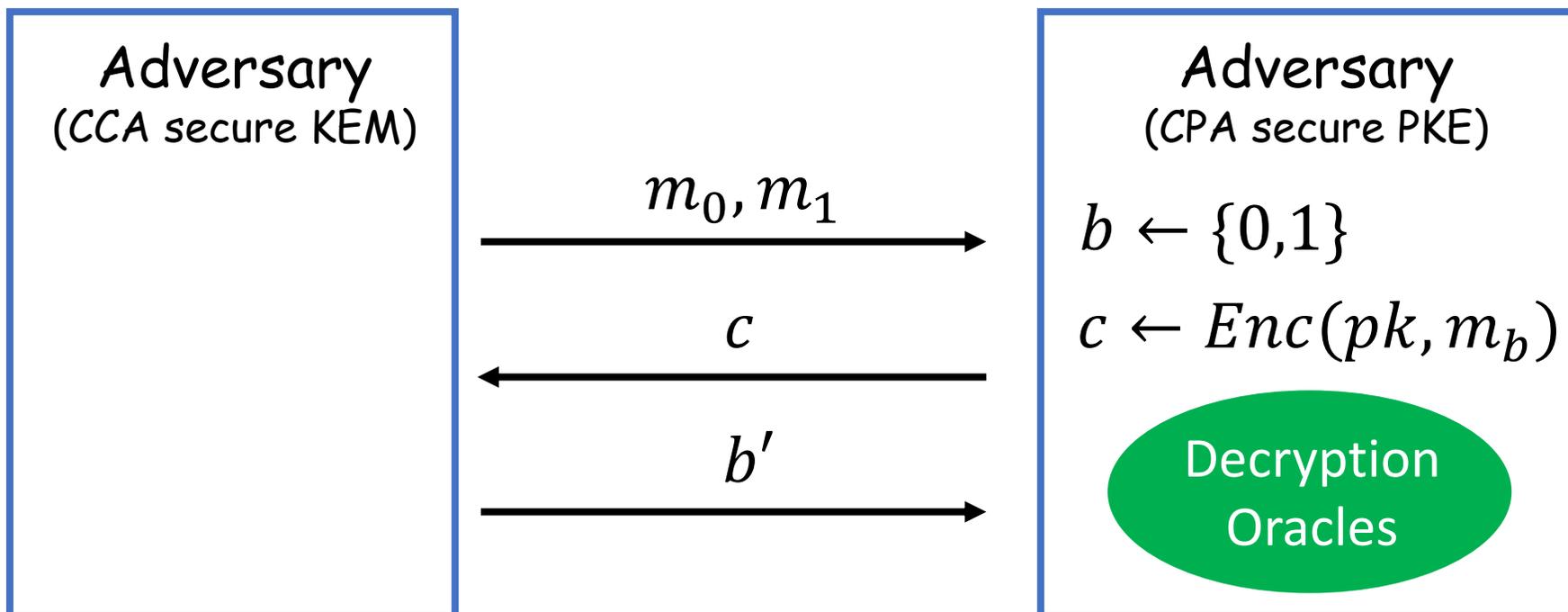
- Fujisaki-Okamoto (FO) transform.



Background

□ CCA-secure Key encapsulation mechanism (KEM)

- Fujisaki-Okamoto (FO) transform.



Background

Lattice based PKE(or KEM)

- Related works

NewHope

RLWE

KYBER

MLWE

SABER

MLWR

RLizard

RLWE+ RLWR

LAC

RLWE

Round5

RLWR

ThreeBears

I-MLWE

Our Goal

Application of PQC-KEM

- TLS Protocol
- IKEv2 Protocol

Performance of lattice based KEM

Algorithm	Time(s)
RLIZARD-ECDSA-WITH-ARIA-128-CBC-SHA256 (RLizard.KEM)	0.012
RLIZARD-ECDSA-WITH-AES-128-CBC-SHA256 (RLizard.KEM)	0.011
NEWHOPE-ECDSA-WITH-AES-128-CBC-SHA256 (NEWHOPE 12289)	0.012
ECDH-ECDSA-WITH-AES-128-CBC-SHA256 (ECDH25519)	0.009

『Development of lattice-based post-quantum public-key cryptographic schemes』
(’20.2.14. / Ewha Womans Univ.)

※ Our goal is to construct **lattice based KEM with short-ciphertext**.

02

Design Rationale



Q. How to construct lattice-based KEM with short-ciphertext?

Design Rationale

□ **LWE(R) vs. Module-LWE(R) vs. Ring-LWE(R)**

- Size of the public key and the ciphertext (byte)

	Public Key	Ciphertext	Ref.
LWE	$n \times \bar{n} \times \log q/8 + Seed_A$	$\bar{m} \times n \times \log q/8 + \bar{m} \times \bar{n} \times \log q/8$	$\bar{n} = \bar{m} = 8$ (FrodoKEM)
MLWE	$n \times k \times \log q/8 + Seed_A$	$k \times n \times \log q/8 + n \times \log q/8$	$k=2,3,4$ (Kyber)
RLWE	$n \times \log q/8 + Seed_A$	$n \times \log q/8 + n \times \log q/8$	-

- **Size of pk & ctx** : $LWE \geq MLWE \geq RLWE$ (depend on parameters)
- **Speed of implementation** : $RLWE \geq MLWE \geq LWE$ (depend on multiplication)

Design Rationale

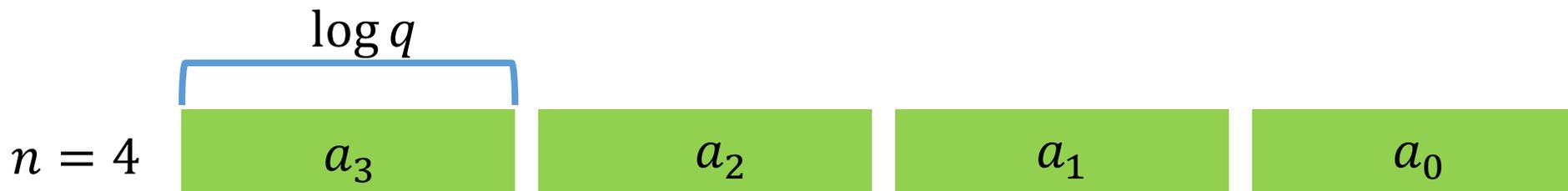
□ **LWE(R) vs. Module-LWE(R) vs. Ring-LWE(R)**

- Size of the public key and the ciphertext (byte)

	Public Key	Ciphertext
RLWE	$n \times \log q / 8 + \text{Seed}_A$	$n \times \log q / 8 + n \times \log q / 8$

$$\mathbb{Z}_q[X] / \langle X^4 + 1 \rangle$$

$$a_3x^3 + a_2x^2 + a_1x + a_0$$



Design Rationale

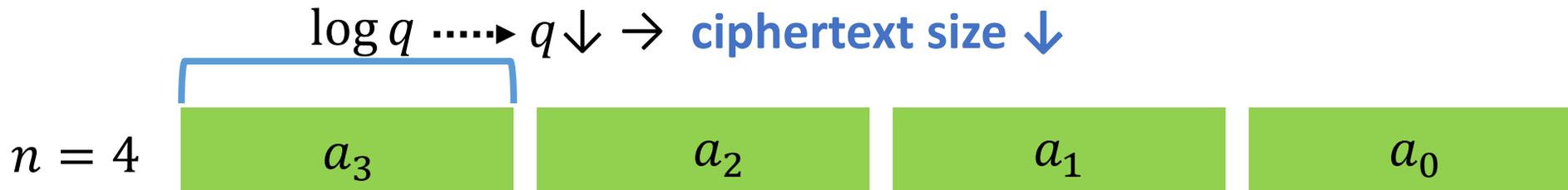
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Design Rationale

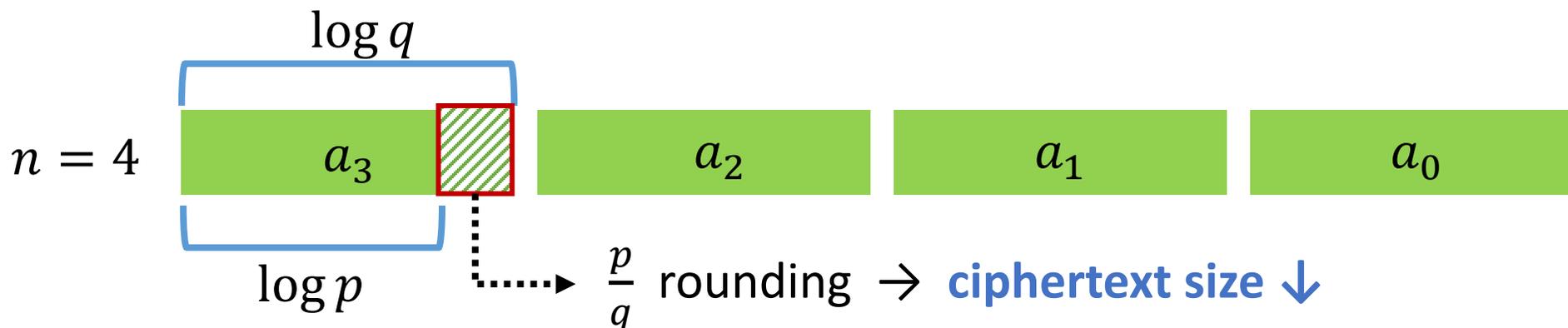
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$$\mathbb{Z}_q[X]/\langle X^4 + 1 \rangle$$

$$a_3x^3 + a_2x^2 + a_1x + a_0$$



Design Rationale

□ only RLWE vs. only RLWR vs. **RLWR+RLWE**

pk

ctx

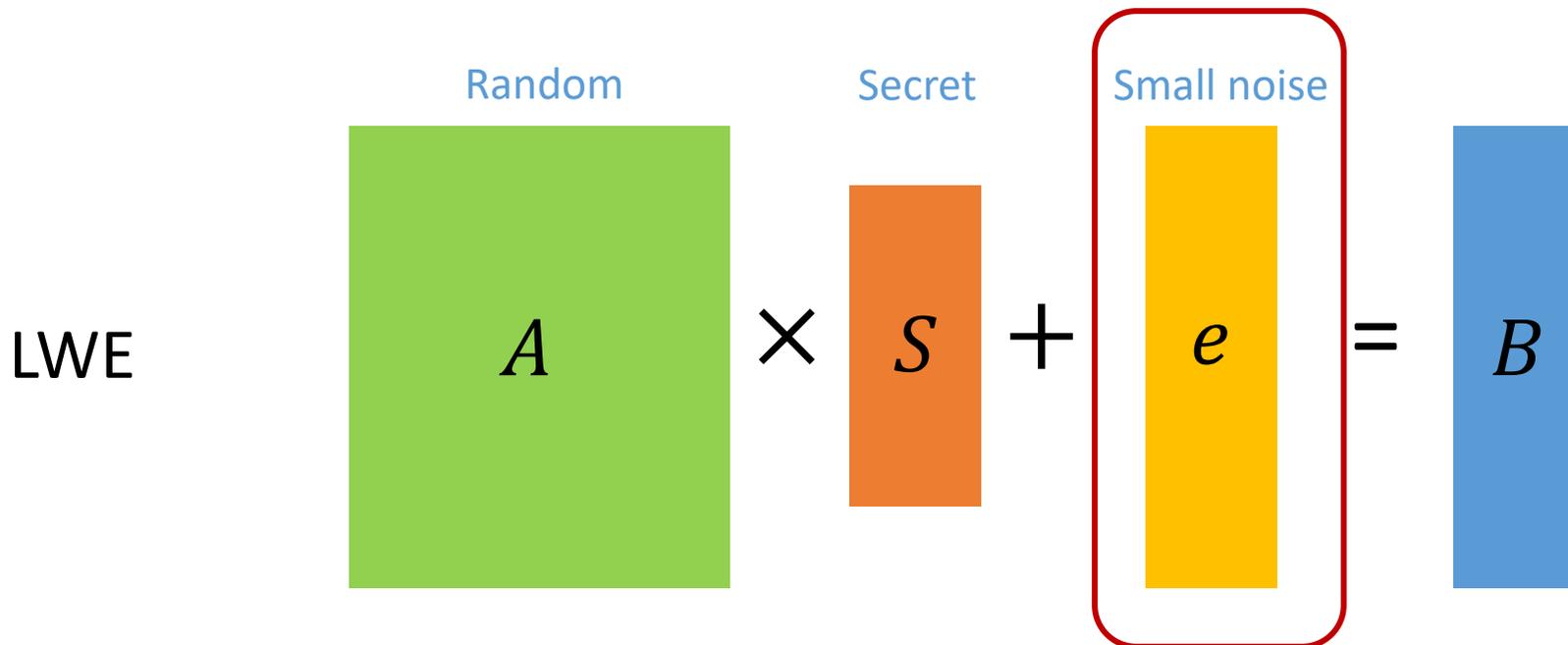
- RLWE + RLWE : To reduce the size of the ciphertext, **the compression function** is needed
- RLWR + RLWR : Parameters setting is difficult & **Decryption failure rate** ↑
- **RLWR + RLWE** : The size of the ciphertext is similar to only RLWR

(Using **the compression function**)

- By adjusting the standard deviation of the noise distribution, difficulties in parameter setting are solved. (& **Decryption failure rate** ↑)

Design Rationale

□ Decryption Failure Rate(DFR)

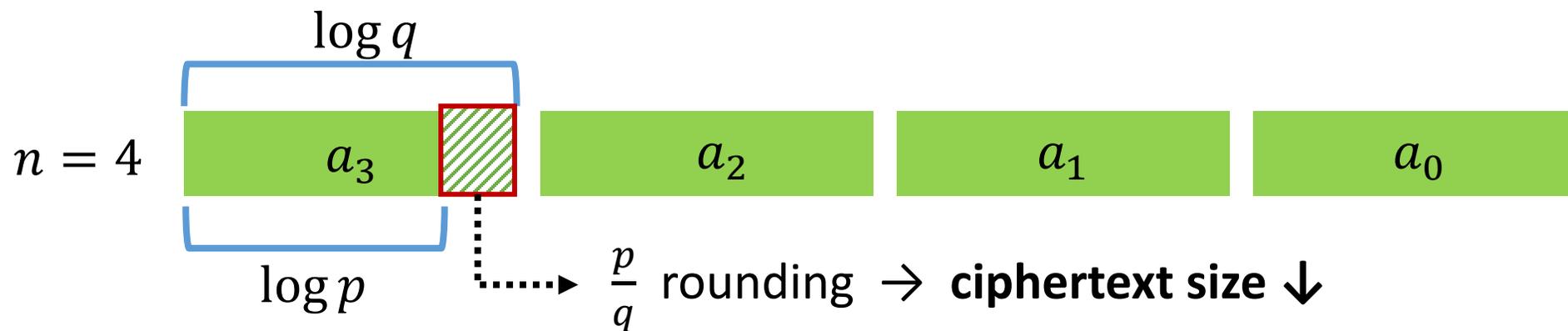


RLWE

$$(a'_3 + e_3)x^3 + (a'_2 + e_2)x^2 + (a'_1 + e_1)x + (a'_0 + e_0)$$

Design Rationale

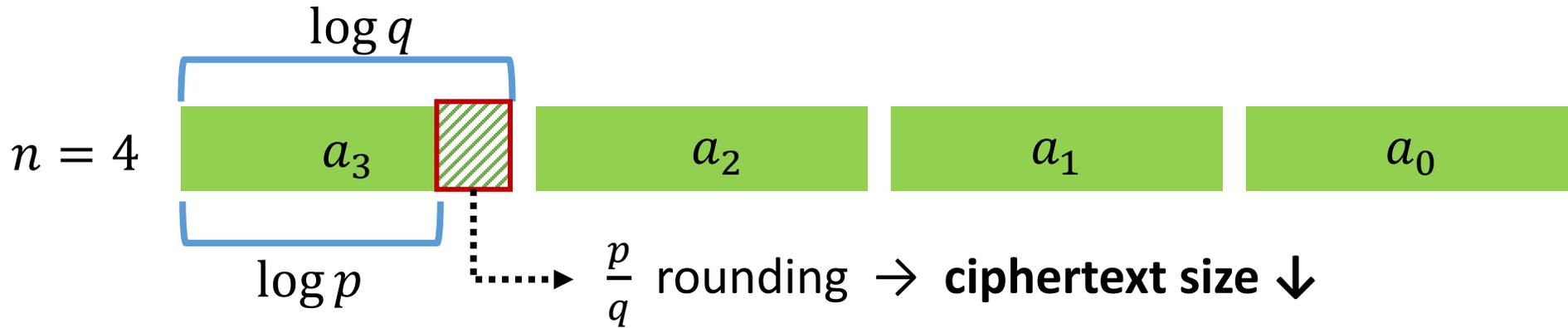
□ Decryption Failure Rate(DFR)



- $q \downarrow \rightarrow$ ciphertext size $\downarrow \rightarrow$ Decryption Failure Rate \uparrow

Design Rationale

□ Decryption Failure Rate(DFR)



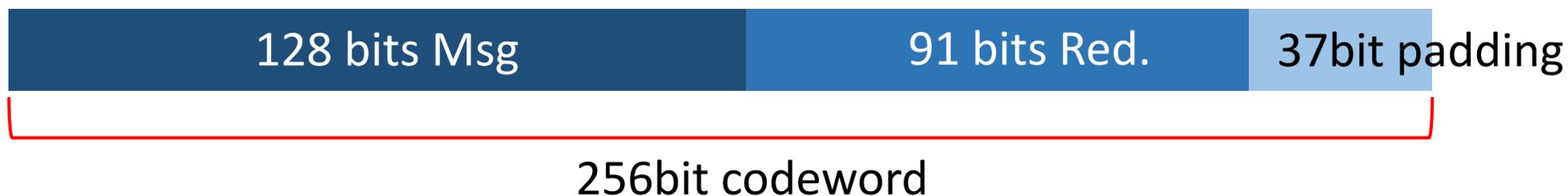
- $q \downarrow \rightarrow$ ciphertext size $\downarrow \rightarrow$ **Decryption Failure Rate \uparrow**
- To solve this problem, we use **error correction codes XEf and D2**.

Design Rationale

□ Error correction code : XEf + D2

- To solve DFR \uparrow , we use error correction codes Xef and D2.
- XEf [Round5] : Efficient implementation (600cycles, 5bits correction)

➤ TiGER 128 : XE3



➤ TiGER 192, 256 : XE5



Design Rationale

□ Error correction code : XEf + D2

- To solve DFR \uparrow , we use error correction codes Xef and D2.
- XEf [Round5] : Efficient implementation (600cycles, 5bits correction)
 - TiGER 128 = XE3 (128bits Msg, 91bit Red, 37bits padding = 256bits codeword)

$n = 512$

256bits codeword

256bits

- TiGER 192, 256 = XE5 (256bits Msg, 234bits Red, 22bit padding = 512bits codeword)

$n = 1024$

512bits codeword

512bits

Design Rationale

□ Error correction code : XEf + D2

- To solve DFR \uparrow , we use error correction codes Xef and D2.
- D2 [Newhope] : Encoding from one message bit to two coefficients
 - ❖ Reduce decryption failure bound from $q/4$ to $q/2$

➤ TiGER 128 : XE3 + D2

$n = 512$

256bits codeword

256bits codeword

➤ TiGER 192, 256 : XE5 + D2

$n = 1024$

512bits codeword

512bits codeword

Design Rationale

□ Description

Public Key : **RLWR**

$SHAKE256(Seed_a, n/8) \rightarrow$  a

$$\left[\left(\frac{q}{p} \right) \right] \left[\text{green box } a \right] * \left[\text{orange box } s \right] \right] = \left[\text{blue box } b \right] \left[\text{hatched box} \right]$$

Ciphertext : **RLWE + Compression**

$$\left[\left(\frac{k_1}{q} \right) \left[\begin{array}{c} \text{green box } a \\ \text{grey box } r \\ \text{yellow box } e_1 \end{array} \right] \right] \rightarrow \left[\text{blue box } c_1 \right] \left[\text{hatched box} \right]$$

XEf & D2

$$\left[\left(\frac{k_2}{q} \right) \left[\begin{array}{c} \text{green box } eccENC(M) + b \\ \text{grey box } r \\ \text{yellow box } e_1 \end{array} \right] \right] \rightarrow \left[\text{blue box } c_2 \right] \left[\text{hatched box} \right]$$

Design Rationale

□ Description

Public Key : **RLWR**

$SHAKE256(Seed_a, n/8) \rightarrow$  a

$$\left[\begin{array}{c} q \\ p \end{array} \right] \left[\begin{array}{c} a \\ * \\ s \end{array} \right] = \left[\begin{array}{c} b \\ \text{hatched} \end{array} \right]$$

Parameters

$R_q := \mathbb{Z}_q[X]/\langle X^n + 1 \rangle$, where n is power of 2.

$n = 512, 1024$ $q = ??$, $p = ??$

$k_1 = ??$, $k_2 = ??$

Ciphertext : **RLWE + Compression**

$$\left[\begin{array}{c} \left(\frac{k_1}{q} \right) \\ * \\ + \end{array} \right] \left[\begin{array}{c} a \\ r \\ e_1 \end{array} \right] \rightarrow c_1 \text{ (hatched)}$$

$$\left[\begin{array}{c} \left(\frac{k_2}{q} \right) \\ * \\ + \end{array} \right] \left[\begin{array}{c} \text{Xef \& D2} \\ ecc(M) + b \\ r \\ e_1 \end{array} \right] \rightarrow c_2 \text{ (hatched)}$$

Design Rationale

□ All integer modulus are **power of 2**

- rounding & ctx compress → ADD & AND operation
- Fixed $q = 256$ is a byte size.
- Efficient modulo operation and memory usage

	Security	n	q	p	k_1	k_2
TiGER128	AES128	512	256	128	64	16
TiGER192	AES192	1024	256	128	64	4
TiGER256	AES256	1024	256	128	128	4

03

Proposed Scheme

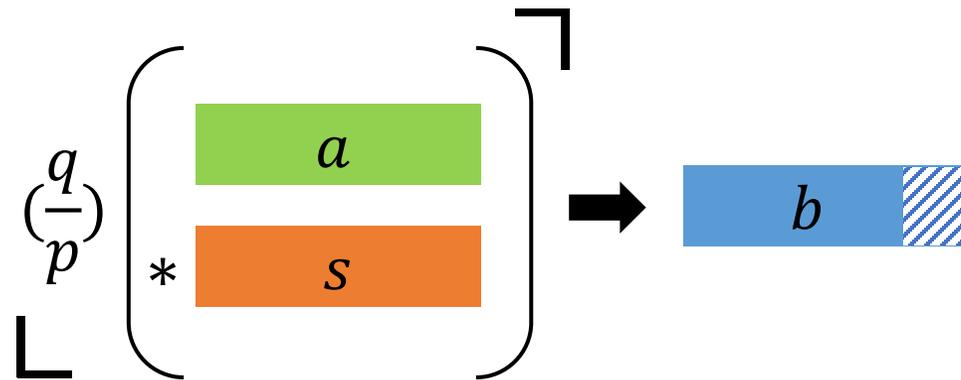


Proposed Scheme

□ PKE_KeyGen

- Input : Security Parameters 1^λ
- Output : pk, sk
 - $\mathbf{a} \leftarrow \text{SHAKE256}(\text{Seed}_a, n/8)$
 - $\mathbf{s} \leftarrow \text{HWT}(h_s, \text{Seed}_s)$
 - $\mathbf{b} \leftarrow \lfloor (p/q) \cdot \mathbf{a} * \mathbf{s} \rfloor$

 - $pk = \langle \text{Seed}_a || \mathbf{b} \rangle$
 - $sk = \langle \mathbf{s} \rangle$

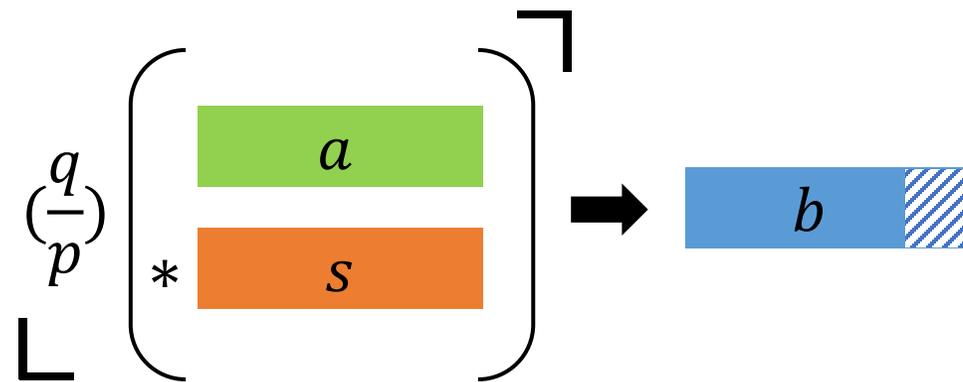


Proposed Scheme

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 - $pk = \langle \text{Seed}_a || \mathbf{b} \rangle$
 - $sk = \langle \mathbf{s} \rangle$



Size of pk (**TiGER128**)

$$n = 512, q = 256, p = 128$$

$$32 + n \cdot \frac{\log(p)}{\log(q)} = 480 \text{ bytes}$$

Proposed Scheme

□ PKE_Encryption

● Input : $pk, M \in \{0,1\}^d$

● Output : c

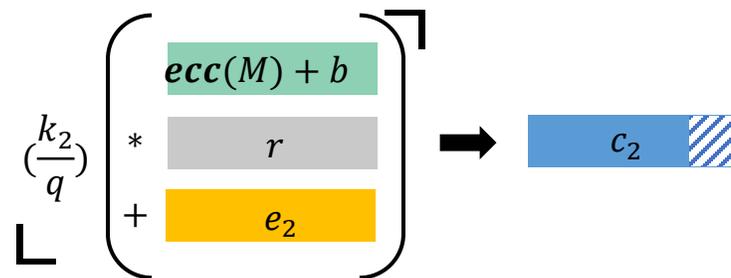
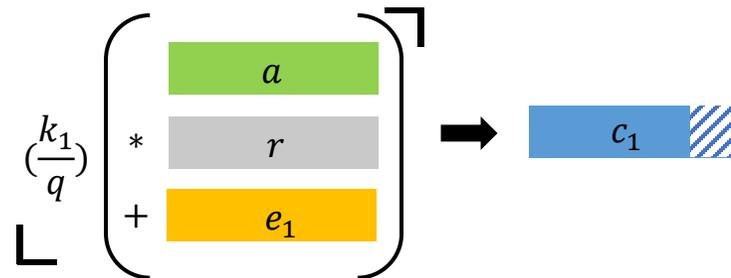
➤ $a \leftarrow \text{SHAKE256}(\text{Seed}_a, 8/n)$

➤ $r \leftarrow \text{HWT}(h_r, w)$

➤ $c_1 \leftarrow \lfloor (k_1/q) \cdot (a * r) + e_1 \rfloor$

➤ $c_2 \leftarrow \lfloor (k_2/q) \cdot \left(\left(\frac{q}{2}\right) \cdot \text{eccENC}(M) + \left(\left(\frac{q}{p}\right) \cdot b\right) * r + e_2 \right) \rfloor$

➤ $c = \langle c_1 || c_2 \rangle$



Proposed Scheme

□ PKE_Encryption

● Input : $pk, M \in \{0,1\}^d$

● Output : c

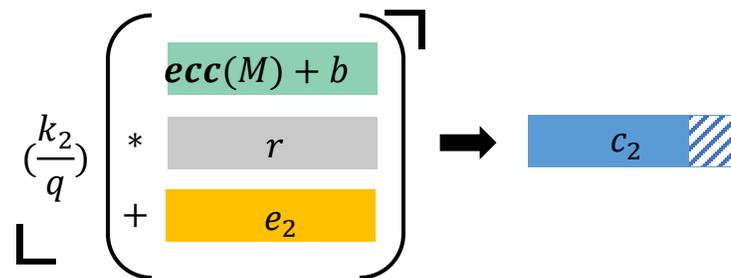
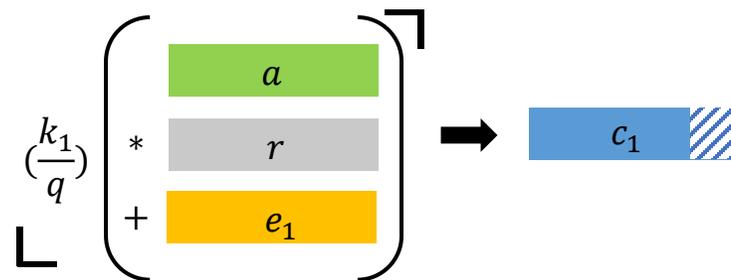
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➤ $c = \langle c_1 || c_2 \rangle$



Size of ctx (TiGER128)

$$n = 512, q = 256, p = 128, k_1 = 64, k_2 = 16$$

$$n \cdot \frac{\log(k_1)}{\log(q)} + n \cdot \frac{\log(k_2)}{\log(q)} = 384 + 256 = 640 \text{ bytes}$$

Proposed Scheme

□ PKE_Decryption

- Input : sk, c

- Output : M

- $\hat{M} \leftarrow \lfloor (2/q) \cdot \left(\left(\frac{q}{k_2} \right) \cdot c_2 - \left(\left(\frac{q}{k_1} \right) \cdot c_1 \right) * s \right) \rfloor$

- $M = eccDec(\hat{M})$

Proposed Scheme

□ KEM_KeyGen

- Input : Security Parameters 1^λ
- Output : pk, sk
 - $pk, sk_{PKE} \leftarrow \mathbf{PKE_KeyGen}(1^\lambda)$
 - $\mathbf{u} \leftarrow R_2$

 - $pk = \langle Seed_a || \mathbf{b} \rangle$
 - $sk = \langle sk_{PKE} || \mathbf{u} \rangle$

Proposed Scheme

□ KEM_Encryption

- Input : pk
- Output : c, K
 - $\delta \in \{0,1\}^d$
 - $c \leftarrow \mathbf{PKE_Encryption}(pk, \delta; H(\delta, H(pk)))$
 -
 - $K = G(H(c), \delta)$

Proposed Scheme

□ KEM_Decryption

- Input : pk, sk, c
- Output : K
 - $\hat{\delta} \leftarrow \mathbf{PKE_Decryption}(sk_{PKE}, c)$
 - $\hat{c} \leftarrow \mathbf{PKE_Encryption}(pk, \hat{\delta}; H(\hat{\delta}, H(pk)))$
 - **if** $c = \hat{c}$ **then** $K \leftarrow G(H(c), \delta)$ **else** $K \leftarrow G(H(c), u)$

Proposed Scheme

□ Parameters and Size of pk , sk , and ctx

Table 1: The detail parameters for each security level

<i>parameters</i>	security level	n	q	p	k_1	k_2	h_s	h_r	h_e	d	f
TiGER128	AES128	512	256	128	64	16	142	110	32	128	3
TiGER192	AES192	1024	256	128	64	4	132	132	32	256	5
TiGER256	AES256	1024	256	128	128	4	196	196	32	256	5

Table 2: Size of pk , sk , and ciphertext (bytes)

<i>parameters</i>	Ciphertext	Public key	Secret key*
TiGER128	640	480	528
TiGER192	1,024	928	1,056
TiGER256	1,152	928	1,056

Proposed Scheme

□ Parameters and Size of pk , sk , and ctx

Table 1: The detail parameters for each security level

<i>parameters</i>	security level	n	q	p	k_1	k_2	h_s	h_r	h_e	d	f
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TiGER192	AES192	1024	256	128	64	4	132	132	32	256	5
TiGER256	AES256	1024	256	128	128	4	196	196	32	256	5

Table 2: Size of pk , sk , and ciphertext (bytes)

<i>parameters</i>	Ciphertext	Public key	Secret key*
TiGER128	640	480	
TiGER192	1,024	928	
TiGER256	1,152	928	

Scheme	ctx	pk
Kyber1024	1568B	1568B
FireSaber	1472B	1312B
LizarMong	1280B	1056B

Proposed Scheme

□ Decryption Failure Rate

- error rate $\hat{\epsilon} = 1 - \Pr[-\frac{q}{2} < \{(e'_br + e'_2 + e_{c2}) - (e'_1s + e'_{c1})\} < \frac{q}{2}]$
- The error rate of each message bit is $2^{-44.28}$ on TiGER128
- Using XEf to correct 3-bits error, decryption failure rate is

$$\epsilon = 1 - \left(\sum_{f=0}^3 \binom{512}{f} \cdot ((2^{-44.28})^f) \cdot (1 - 2^{-44.28})^{512-f} \right) \approx 2^{-145.75}$$

	Bit error rate	DFR	f
TiGER128	$2^{-44.28}$	$2^{-145.75}$	3
TiGER192	$2^{-33.48}$	$2^{-150.41}$	5
TiGER256	$2^{-41.96}$	$2^{-201.29}$	5

04

Security



Security

- **Theorem 1 (IND-CPA PKE).** *The above PKE scheme is secure under chosen plaintext attacks if the RLWE assumption and the RLWR assumption holds. That is, for any PPT adversary \mathcal{A} , we have that $\mathbf{Adv}_{PKE}^{IND-CPA}(\mathcal{A}) \leq \mathbf{Adv}_{n,q}^{RLWE}(\mathcal{B}) + \mathbf{Adv}_{n,q,p}^{RLWR}(\mathcal{B})$.*

$$pk = \langle Seed_a || \mathbf{b} = \lfloor (p/q) \cdot \mathbf{a} * \mathbf{s} \rfloor \rangle$$



Decisional **RLWR** problem

$$pk = \langle Seed_a || \mathbf{b} \leftarrow \mathbf{R}_p \rangle$$

In the random oracle model, $\mathbf{a} \leftarrow H(Seed_a)$.

Security

□ **Theorem 1 (IND-CPA PKE).** *The above PKE scheme is secure under chosen plaintext attacks if the RLWE assumption and the RLWR assumption holds. That is, for any PPT adversary \mathcal{A} , we have that $\mathbf{Adv}_{PKE}^{IND-CPA}(\mathcal{A}) \leq \mathbf{Adv}_{n,q}^{RLWE}(\mathcal{B}) + \mathbf{Adv}_{n,q,p}^{RLWR}(\mathcal{B})$.*

$$\mathbf{c}_1 \leftarrow \lfloor (k_1/q) \cdot (\mathbf{a} * \mathbf{r}) + \mathbf{e}_1 \rfloor$$

$$\mathbf{c}_2 \leftarrow \lfloor (k_2/q) \cdot \left(\left(\frac{q}{2} \right) \cdot eccENC(M_b) + \left(\left(\frac{q}{p} \right) \cdot \mathbf{b} \right) * \mathbf{r} + \mathbf{e}_2 \right) \rfloor$$



Decisional **RLWE** problem

$$\mathbf{c}_1 \leftarrow \lfloor (k_1/q) \cdot \mathbf{u} \rfloor$$

$$\mathbf{c}_2 \leftarrow \lfloor (k_2/q) \cdot \left(\left(\frac{q}{2} \right) \cdot eccENC(M_b) + \mathbf{v} \right) \rfloor$$

Security

□ **Theorem 2 (IND-CCA KEM in QRROM).** *We define a public key encryption scheme $PKE = (\text{KeyGen}, \text{Encrypt}, \text{Decrypt})$ with message space \mathcal{M} and which is $(1-\epsilon)$ -correct. For any IND-CCA quantum adversary \mathcal{A} that makes at most q_D queries to the decryption oracle, at most q_G queries to the random oracle G and at most q_H queries to the random oracle H , we have that*

$$\mathbf{Adv}_{KEM}^{IND-CCA}(\mathcal{A}) \leq 2q_H \frac{1}{|\mathcal{M}|} + 4q_G \sqrt{1-\epsilon} + 2(q_G + q_H) \sqrt{\mathbf{Adv}_{PKE}^{IND-CPA}(\mathcal{B})}.$$

Security

□ Analysis of known attacks (using LATTICE-ESTIMATOR[ASP15])

- Core-SVP [ADPS16] & Meet-Attack [MAY21]

	TiGER Core-SVP	Kyber Core-SVP	NIST req.
AES128	129	118	143
AES192	231	183	207
AES256	261	256	272

- MATZOV [MAT22]

	TiGER MATZOV	Kyber MATZOV	NIST req.
AES128	147	140	143
AES192	246	201	207
AES256	277	270	272

Performance

□ Performance(CPU cycles)

- 2~2.4x faster than Kyber(ref), 2.6~4.0x faster than LAC(opt)

Algorithm	Key generate	Encapsulation	Decapsulation
TiGER128 (ref)	58,531	74,312	97,258
TiGER192 (ref)	72,661	128,854	151,382
TiGER256 (ref)	88,441	159,665	193,663
Kyber512 (ref ³)	121,721	153,724	189,515
Kyber768 (ref)	217,175	261,818	304,349
Kyber1024 (ref)	308,615	353,579	411,223
LAC128 (opt ⁴)	138,841	219,415	253,301
LAC192 (opt)	308,557	414,122	638,422
LAC256 (opt)	368,792	595,165	806,561
Kyber512 (AVX2 ⁵)	34,672	47,670	41,675
Kyber768 (AVX2)	59,150	73,523	64,653
Kyber1024 (AVX2)	92,268	121,576	106,296

✓ **Implementation**
AMD Ryzen3 2200G@3.5GHz,
Ubuntu 22.04.1,
GCC 11.3.0 with -O3
Keygen : 100,000
Enc/Dec : 100,000

Conclude

- **TiGER : Tiny bandwidth KEM for easy miGration based on RLWE(R)**
 - **Keygen: RLWR, Enc • Dec: RLWE / $q=256$, using ECC ($XE_f + D_2$)**
 - Short Public Key and Ciphertext
 - **Achieve the security level** AES128, AES192, and AES256
 - **Fast and suitable for SIMD.**



An Optimal KEM for Quantum Resistant Security Protocols



Q&A

THANK YOU!

